Name: ______________________

MA 1118 - Multivariable Calculus
Quiz 2 - Quarter I - AY 02-03

Instructions: Work all problems. Read the problems carefully. Show appropriate work, as partial credit will be given. No notes or tables permitted.

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1. (20 points) Determine whether each of the following series converges absolutely, converges conditionally, or diverges. (Identify the test(s) used in each case):

(a.) $$\sum_{n=0}^{\infty}(-1)^n \frac{n^2+5}{n^4+1}$$

solution:

Observe that this is an alternating series, but also

$$a_n = (-1)^n \frac{n^2+5}{n^4+1} \qquad \Longrightarrow \qquad |a_n| = \frac{n^2+5}{n^4+1} \ \rightarrow \ \frac{n^2}{n^4} = \frac{1}{n^2}$$

for "large" n. Therefore, the given series converges absolutely by the limit comparison test and p-tests ($p = 2$).

(b.) $$\sum_{n=1}^{\infty}\frac{n!\ 3^n}{10^n}$$

solution:

Observe that this is a positive term series, with both factorials and variable exponents. This strongly suggests we try the ratio test, i.e.

$$a_n = \frac{n!\ 3^n}{10^n} \qquad \Longrightarrow \qquad \frac{a_{n+1}}{a_n} = \frac{\frac{(n+1)!\ 3^{(n+1)}}{10^{(n+1)}}}{\frac{n!\ 3^n}{10^n}}$$

Therefore

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)!}{n!}\ \frac{3^{(n+1)}}{3^n}\ \frac{10^n}{10^{(n+1)}} = \frac{3}{10}(n+1) \quad \rightarrow \quad \infty$$

as $n \rightarrow \infty$. Therefore, the series diverges by the ratio test.

(c.) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{1+n}$

solution:

Observe that this is a positive term series, and

$$a_n = \frac{\sqrt{n}}{1+n} \quad \to \quad \frac{\sqrt{n}}{n} = \frac{1}{\sqrt{n}}$$

Therefore, since $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges by the p-test with $p = 1/2$, then the original series diverges by the limit comparison and p-tests.